CET 2B Example Sheet 1 (The final one)

Rheology

Sections 1 and 2.

1. a) Using a time dependant 'Cross' structure model. Determine the shear stress of a fluid that has been subjected to a constant shear rate $\dot{\gamma}$ = 5 s^{-1} for a period of 5 s. Assume that the fluid started ***from rest*** with a viscosity of 3 Pas. Data for model given below.

b) If, after the initial 5 s at $\dot{\gamma}$ = 5 s^{-1}, the fluid is not sheared for a further 15 s, determine the fluids viscosity after this time. Following the 15 s no shear period, the fluid is sheared again at 20 s^{-1}, determine the shear stress after a period of 10 s at this shear rate. Draw sketches of the time evolution of the shear rate, viscosity and stress for the fluid subject to these boundary conditions.

Data: η_o = **3 N s/m^2**

η_∞ = **0.1 N s/m^2**

n = **0.6**

k_1 = **0.02 s^{-1}**

k_2 = **0.016 s^{-1}**

Answers:

τ_5 = **11.77 N/m^2**

η_{20} = **2.49 Pas**

τ_{30} = **19.28 N/m^2**

c) The Cross structure model is one of a number of models that can describe Non Newtonian shear thinning behaviour. Write down the mathematical formulation of others and show how they differ in terms of a flow curve response. Explain the physical significance, if one exists, for each of the models.

Now do Tripos Question without looking at solution!

06.9.1

2. Fluid flows as a steady, two dimensional (x,y) laminar flow within a

"Semi infinite" parallel walled rectangular section duct. The length of the duct x is 0.5 m, The duct width y, across which a transverse velocity profile develops, is 4 mm and the depth of the duct z is 50 mm. z=50>>y=4 so the flow approximates to 2D.

The fluid has been modelled in three ways.

(a)	Newtonian	η	=	1.3 N s/m^2
(b)	Power Law	k	=	40 N s$^{0.8}$/m^2
		n	=	0.2
(c)	Bingham	η	=	0.3 Ns/m^2
		τ_y	=	100 N/m^2

Derive expressions and sketch the velocity profiles for each of the above constitutive equation flow. Determine the volumetric flow rates predicted by each equation for a pressure drop of 0.3 bar along the length of the duct and determine the times for breakthrough (the time for the first part of an initially uniform pulse across the duct depth) to reach the end of duct) of a tracer and the mean residence time of the fluid in each case. Sketch the form of the residence time distribution that you would expect for each fluid.

Answers:	**Newtonian**	**1.23**	**x**	**10^{-5} m^3/s**
	Power Law	**1.39**	**x**	**10^{-5} m^3/s**
	Bingham	**2.08**	**x**	**10^{-6} m^3/s**

3. Two fluids flow within a tube of radius r_o = 5 mm. In the central region of the tube r = 0 to r = 4 mm a Newtonian fluid, **A,** is flowing as a core of fluid. Within the annular region r = 4 mm to r_o = 5 mm, a Power Law fluid **B** is flowing in the same direction as fluid A.

Given a pressure gradient of, 5 x 10^4 Pa/m along the tube, determine,

a. The maximum velocity within the tube,
b. The velocity at the A/B interface
c. The volumetric flow rate.

Assume no slip boundary conditions at the tube walls and stress and velocity continuity across the A/B interface.

Data.

Viscosity of fluid A. 10 Pas

Fluid B. $\tau = 10\ \dot{\gamma}^{0.5}$

Answers U_{max} **= 0.147 m/s,** $U_{A/B}$ **= 0.127 m/s Q = 8.751 x 10** $^{-6}$ **m³/s**

Now do Tripos questions,
02.4.7, 03.4.04, 04.4.4. 06.9.2, 10.9.1
Don't look at solutions until you have done them!

4. Write short notes on the following;

a) Why is water Newtonian, whereas a molten polyethylene is Non Newtonian?

b) Your blood shear thins; Why?

c) Molten polypropylene shear thins; Why?

d) Do fluids really have a yield stress as modeled by the Bingham equation?

e) Discuss the strength and weakness of modeling molten polymers using a power law equation and the strength and weakness of modeling tomato Ketchup using a Bingham equation.

f) Choose the most appropriate Non Newtonian constitutive equation for the following fluids and give reasons for your decision. Give estimates of either the zero shear rate viscosity or where appropriate the yield stress for each fluid.

i)	Timotei shampoo
ii)	Heinz Tomato Ketchup
iii)	Potty putty
iv)	Robinson College consume soup
v)	North sea gas at 200bar, 20 centigrade
vi)	Kings College thick English broth
vii)	Axial grease
viii)	Lubricating oil
ix)	Honey
x)	Sainsbury yoghurt

xi) A Non Newtonian fluid of your own choice
xii) Your own blood
xiii) Ink jet printer fluid

Now Take a break and have your Non Newtonian first supervision !

Section 3

5. Derive from first principles (without using your notes!) the constitutive equation for a 'Maxwell fluid' expressed in its

(a) Differential form
(b) Integral form, as a function of past strain rate
(c) Integral form, as a function of past strain

Using each of the above forms of equation, determine the response for both steady shear and stress relaxation after steady shear. Comment on the results.

What is the steady shear stress and apparent viscosity for a Maxwell fluid with the moduli specified below, at (a) $\dot{\gamma}$ = 10 s^{-1} and (b) $\dot{\gamma}$ = 100 s^{-1}. Comment on your result.
Answer **1.39 10^5 Pa, 1.39 10^6 Pa, 1.39 10^4 Pas**

What is the stress relaxation after steady shear for (a) $\dot{\gamma}$ = 10 s^{-1} and (b) $\dot{\gamma}$ = 100 s^{-1}, at t = 5 x 10^{-1} s and t = 5 s after shear cessation

Data:

g_i	(N/m^2).	3 x 10^5,	5 x 10^4 ,	2 x 10^3,	1 x 10^3,
λ_i	(s).	10^{-3} ,	3.2 x 10^{-2} ,	1 ,	10,

Answers

At t=0.5s **1.07 10^5 Pa**
1.07 10^6 Pa

At t= 5s **6.08 10^4 Pa**
6.08 10^5 Pa

6. A viscoelastic fluid obeys the following constitutive equation

$$\tau(t) = -\int_{-\infty}^{t} \frac{g}{\lambda} e^{-(t-t')/\lambda} \gamma(tt')dt'$$

where g = 10^3 Pa and λ = 2 s.

The fluid starts from a zero stress at time t'= 0 and is then subjected to a strain rate $\dot{\gamma}$ given by

$$\dot{\gamma} = 10\ t', \quad s^{-1}$$

where t' is the time that has evolved from the start time t'=o.
Determine the stress after a time t = 6 s.
Answer: (big calc!) τ = 82.0 **kPa**

7. Write short notes on the following

i) All I need to know is the differential Maxwell equation, there is no difference in formulating the integral form, its more complex and a waste of time.
ii) The integral Maxwell equation is the answer to all non linear viscoelastic problems
iii) Why are the viscoelastic properties of water and low viscosity fluids not particularly important in terms of most engineering problems? Why is the viscoelasticity of printing inks important even though the zero shear viscosity is low?

iv) You are applying for a job with Sainsburys; explain to the non technical interviewer why Sainsburys need to have in house expertise on viscoelastic properties.

Now do Tripos

02.4.8, 03.4.5, 04.4.5, 06.9.3, 10.9.3

Remember, Tripos questions are part of the supervision.

Now have your viscoelastic second supervision

Section 4 of Example Sheet

8. A stress matrix, in a laboratory co-ordinate frame, for a flowing polymer is given below in MPa.

$$\begin{vmatrix} \mathbf{0.5} & \mathbf{0.1} & \mathbf{0} \\ \mathbf{0.1} & \mathbf{0.3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.2} \end{vmatrix}$$

Consider a new co-ordinate frame where the x_1 axis is rotated 30° anticlockwise in the plane x_1, x_2.

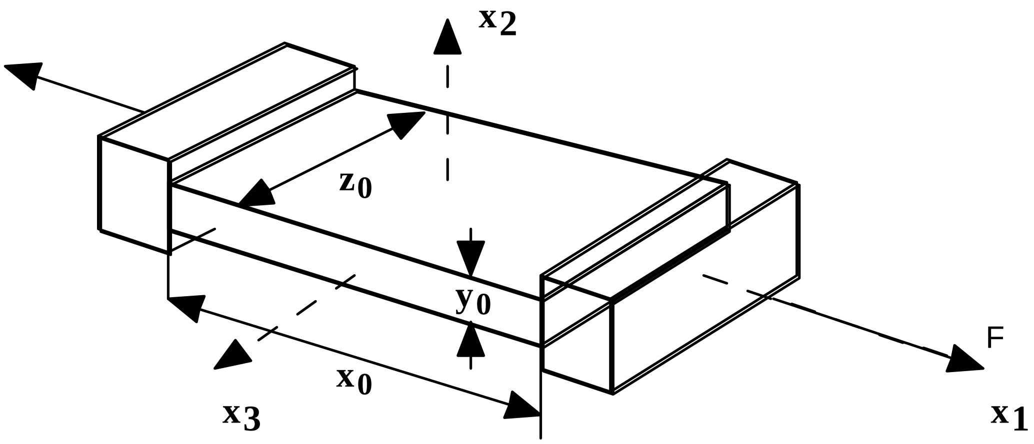
(i) Determine the direction cosine matrix between the new and old laboratory frame.
(ii) Determine the full stress matrix in the new co-ordinate frame.
(iii) Show that both the first and second invariants of the stress matrix are the same for both co-ordinate frames.

Ans $I_1 = 1$, $I_2 = 0.30$

9.
(a) Derive a relation between the extensional strain rates $\dot{\varepsilon}_{11}, \dot{\varepsilon}_{22}, \dot{\varepsilon}_{33}$ for an incompressible fluid.

(b) Write down the strain rate matrix for a 2D pure shear rate deformation.

(c) Using the generalised definition of Newtonion viscosity, derive a relation for extensional viscosity in pure shear.



(d) A strip of molten polymer having initial dimensions x_o = 3 mm y_o = 1 mm z_0 = 5 mm is held in a device shown in the diagram. The polymer is then stretched in the x_1 direction and data relating to the extension and tensile force F is given below.

(i) show that the deformation corresponds to a constant extension rate $\dot{\varepsilon}_{11}$ and determine the magnitude of $\dot{\varepsilon}_{11}$.

(ii) Show that the data is consistent with the polymer behaving as a Newtonian fluid and determine both the extensional and shear viscosity of the polymer. Assume that the z_0 dimension does not change during deformation and that the polymer is incompressible.

Time	S	0	0.5	1.0	1.5
Displacement of moving end	mm	0	4.9	8.15	13.4
Force N	N	0	$6.1\ 10^{-2}$	$3.7\ 10^{-2}$	$2.2\ 10^{-2}$

10. A narrow gap Couette apparatus is used to measure the flow properties of a polymer solution. Using optical techniques it was possible to establish, for a certain flow condition, that the direction of the Principal axis was at an

angle of 15^0 to the direction of flow, in the plane of shear of the flow. The magnitude of the Principal Stress Difference was 30 kPa.

a) If the Couette gap is 1 mm, the inner rotor diameter 30 mm, the depth of fluid 80 mm; determine the torque that would be recorded on the rotor inner cylinder. Use tensor transformation to calculate the shear stress.

b) If the outer cylinder is static and the inner cylinder rotates at an angular velocity of 300 $rads^{-1}$, determine,

i) The apparent shear viscosity of the fluid.

ii) The Normal Stress Difference of the fluid.

11. Write brief notes to test your broader understanding of rheology and processing.

a) Describe the useful rheological features of six different complex fluids that occur either during processing or as a product. In each case identify the most appropriate model to describe the fluids properties.

b) Identify three situations where modeling a complex fluid flowing within a complex flow would/could add value to a Company process.

c) Describe why tensorial aspects of Stress, strain and Strain rate need to be considered when considering **both** complex fluids and complex flows.

Now do

2004 paper 4 Question 6, also 09.9.2 and 10.9.3 again without looking at the solutions.

If you can do them, then you have mastered the course. Good luck for the future and I really hope you don’t come to a sticky end.

Now have your third and final generalized deformation supervision.